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Nonstationary Random Parametric Vibration in Light Aircraft Landing Gear

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In this work, a new approach for analysis of random vibration in light aircraft landing gear for a given duty cycle is developed and studied. The aircraft is modeled as a linear, single-degree-of-freedom oscillator with random properties, including nonstationary damping and random nonstationary load. Note that this type of problem is difficult to analyze efficiently using most conventional techniques. Two approaches to analyze the random vibration of the system are examined: a new variant of the random matrix approach, a statistical random vibration analysis method developed previously by the authors; and a hybrid Monte Carlo technique containing a spectral representation approach and a variant of Latin hypercube sampling. Random response results are shown for two light aircraft landing on three different terrain types using each method, and comparisons are offered. These results show that Monte Carlo analysis cannot compute accurate solutions for this problem. It is anticipated that the proposed random matrix technique could be used in conjunction with current fatigue analysis methods so that accurate landing gear fatigue information may be computed.

Nomenclature

$[A]$ = random property matrix
a = aircraft horizontal acceleration
C = ground spectral density coefficient
$C_{L,\alpha}$ = lift curve slope
c = total damping
c_s = strut and tire damping
c_w = wing damping
E = Young's modulus of strut material
f = external loading on aircraft
g = gravitational acceleration
I = second area moment of strut cross section
k = strut stiffness
L = strut ground length
m = mass of aircraft
N = number of discrete wave numbers used, ground spectral density exponent
$\{\boldsymbol{p}\}$ = random load vector
S = wing area
S_g = ground height spectral density
S_x = deflection spectral density
s = ground distance from touchdown
T = total landing time
t = time
t_d = brake delay time
V = aircraft ground speed
V_d = aircraft descent velocity
V_w = wind speed
V_0 = initial aircraft ground speed
X = displacement amplitude
x = relative displacement function
$\{x\}$ = relative displacement vector
x_g = ground surface height
x_0 = initial relative displacement
β = lift-to-weight ratio
β_0 = initial lift-to-weight ratio
γ = strut and tire structural damping factor
Δt = time step size
$\Delta\kappa$ = wave number spacing
θ = vertical strut angle
κ = wave number
μ = ground friction coefficient
ρ = air density
ϕ = random phase angle
ω = frequency

Introduction

RANDOM vibration of aircraft landing gear, even in light aircraft, is important because random vibration analysis is needed to conduct accurate fatigue analysis. Fatigue analysis and testing of landing gear is used both for aircraft life estimates and for inspection schedules. If the estimated fatigue damage is significantly incorrect, then either safety problems may result or the aircraft may be retired earlier or be inspected more often than necessary. More importantly, it may be desirable to design an aircraft for improved fatigue performance, and thus it becomes necessary to estimate fatigue life of a new aircraft from early design stages. The random vibration analysis techniques offered in this work would allow designers to estimate landing gear vibration environments, so that accurate fatigue life may be computed, and the landing gear design may be changed early and inexpensively if needed.

Analysis of deterministic vibration in light aircraft landing gear is fairly simple. The two aircraft analyzed in this work have landing gear composed of leaf spring struts, which act as linear springs if small deflection is assumed. The damping in

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This work is dedicated to Dr. Constantinos S. Lyrintzis, Associate Professor at San Diego State University's Department of Aerospace Engineering and Engineering Mechanics, who was tragically slain on August 15, 1996. This paper represents his last direct contribution to aerospace engineering. In addition, Dr. Lyrintzis will have a profound indirect impact on the future of engineering through the students and colleagues whose lives he touched. His wisdom, ebullience, and compassion will be sorely missed.

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the system comes from the struts and tires when deflecting, and from the wings when the vertical velocity (with respect to the air) changes; this damping happens to be linear for relatively high ground speed. The external loading on the system results from weight, lift, and the surface profile. Note that the ground speed affects damping and lift and is needed to determine the ground height at the aircraft wheels as a function of time from the height as a function of space. Thus, if an aircraft is accelerating, e.g., during landing or takeoff, both the effective load and the aerodynamic damping become functions of time, making the problem more difficult.

When examining landing gear vibration, it may be more meaningful to consider the properties and loading to be random. For example, the mass of the aircraft, fuel, passengers, and cargo may vary appreciably from one duty cycle, i.e., a takeoff or landing, to another. As mentioned in the preceding text, effective damping depends on the ground speed, which can be considered a random process. Also, the ground surface is a random field varying in space; the ground height at the aircraft wheels may then be expressed as a random process in terms of the aircraft's ground speed. Finally, the initial descent velocity of the aircraft is a random variable. Thus, a vibration analysis of an aircraft duty cycle should take into account random initial conditions, nonstationary random loading, and random, nonstationary time-varying properties; such a vibrating system is said to undergo nonstationary random parametric vibration and is very difficult to analyze.

Most early work in random parametric vibration analysis is well summarized in a monograph by Ibrahim[1] and uses Markov methods based on Ito stochastic calculus or the Fokker–Planck–Kolmogorov equation. However, these methods are limited to systems with load and properties that are noise or filtered noise, and they often yield an infinite hierarchy of equations that can only be solved with some difficulty. Another statistical method mentioned in Ibrahim[1] is stochastic averaging, which assumes that the response process will be nearly harmonic with an amplitude and phase angle that varies slowly with time. Stochastic averaging is useful for a system with small variations in stiffness and nearly harmonic loading, but many problems of interest simply do not meet these criteria. Therefore, none of these early methods are entirely suitable for the analysis of general linear systems undergoing random parametric vibration.

Current techniques that could possibly be used for random parametric vibration would come from stochastic finite element analysis.[2–4] Stochastic finite elements allow for static or dynamic analysis of linear or nonlinear systems with deterministic load and random, time-invariant, spatially dependent properties. An interesting statistical stochastic finite element technique proposed by Ghanem and Spanos[5] uses a Karhunen–Loeve expansion of the property spatial covariance eigenfunctions, and it works well for properties that have noise or filtered-noise spatial correlations. However, this technique is difficult to extend to systems with stochastic loading or time-dependent properties. In general, stochastic finite element techniques have this limitation, and thus they are not well-suited to analysis of nonstationary random parametric vibration.

A current approach that has been used to successfully analyze stationary random parametric vibration problems is the Monte Carlo simulation. Monte Carlo methods represent a random variable by a set of equally weighted deterministic values, allowing relatively fast and simple analysis. The heart of any Monte Carlo method is the sample generation scheme, and several such schemes have been used for various purposes. For example, the autoregressive moving average method[6,7] and the spectral representation method[8,9] can simulate stationary Gaussian random fields or processes, while Latin hypercube sampling[10–12] can simulate any number of correlated variables of any type. Indeed, Seya et. al.[13] employed Latin hypercube sampling for building property simulation while using the spectral representation method to generate sample seismic load histories, illustrating that sample generation schemes may sometimes be combined. Monte Carlo simulation may be a good choice to analyze nonstationary random vibration.

Another technique that may do well in analysis of nonstationary random parametric vibration was developed by Huntington and Lyrintzis[14] to obtain response statistics in stationary random parametric vibration problems. The technique, known as the random matrix method, formulates the random vibration problem in terms of a random matrix equation in the frequency domain. A truncated Neumann expansion is used to obtain statistics of the inverse matrix from the statistics of the original matrix. Huntington and Lyrintzis[14] used this technique to parametrically study the response of a single-degree-of-freedom system with random stationary loading and random, time-varying stationary stiffness.[15] Thus, the random matrix approach may be a good technique to handle the landing gear vibration problem presented here.

The aircraft and landing gear in this work will be modeled as a linear, single-degree-of-freedom system with random initial conditions, random nonstationary time-varying properties, and random nonstationary loading. As mentioned previously, variations of the random matrix approach and Monte Carlo simulation will be presented, and advantages and disadvantages for each method will be discussed. Using the proposed techniques, response statistics will be determined for two aircraft landing on three different terrain types, and comparisons will be offered. Finally, conclusions will be drawn as to the importance of random property modeling in this problem and the utility and accuracy of the proposed techniques.

Aircraft Model

The undeformed geometry of a leaf spring strut used in light aircraft landing gear is shown in Fig. 1. Each strut has a rectangular cross section and is uniform along its length; the stiffness of a single strut can be found from small deflection beam theory, assuming that the strut is a cantilever beam affixed to the fuselage. The total stiffness of both struts is given by

$$k = (6EI/L^3)\sin(\theta) \tag{1}$$

The mass of the aircraft can of course be derived from its weight (which is assumed to be time-independent):

$$m = W/g \tag{2}$$

The damping in the system comes from two sources: 1) structural damping in the tires and struts, and 2) aerodynamic damping from the wing (which is caused by changes in effective angle of attack from the aircraft's vertical velocity). The total damping is given by

$$c = \gamma\sqrt{km} + \tfrac{1}{2}C_{L\alpha}\rho(V + V_w)S \tag{3}$$

Equation (3) is valid only when the aircraft's airspeed is much higher than its vertical velocity. This condition is true for the aircraft landing problems studied in this analysis.

In fatigue analysis, we are interested in the relative displacement of the aircraft with respect to the ground. This means that the load term will have a contribution from the external

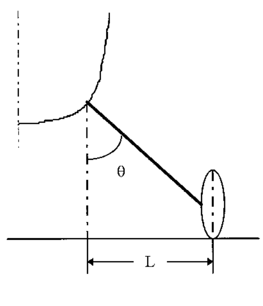

Fig. 1 Geometry of a leaf spring landing gear strut used in this work.

force on the aircraft (weight and lift) and a contribution from the x_g. This effective load becomes

$$f = W\left[1 - \beta_0\left(\frac{V + V_w}{V_0 + V_w}\right)^2\right] - mx_g'(s)a - mx_g''(s)V^2 - \tfrac{1}{2}\rho SC_{L,\alpha}x_g'(s)V^2 \quad (4)$$

where the primes denote spatial derivative. This loading function assumes that the aircraft is always in contact with the ground, which will not always be the case in an actual landing. However, to ensure that the analyzed system is linear, this assumption is required for the vibration analysis. Finally, the initial displacement is zero, because $t = 0$ represents touchdown, and the initial downward velocity is a random variable.

Some points should also be made regarding the ground height field. The spectral density for this field is assumed to have the following form[16]:

$$S_{x_g}(\kappa) = C\kappa^{-N} \quad (5)$$

where the exponent $N = 2.1$ for most landing surfaces; the spectral density for ground slope can be obtained by multiplying $S_{x_g}(\kappa)$ by κ^2, and the ground curvature spectral density is $S_{x_g}(\kappa)\kappa^4$. There are potential problems with this spectral density at both low wave number and high wave number. At very low wave number, the spectral density for the ground height and slope approach infinity, which is not realistic; this difficulty is minimized in a discrete wave number domain by setting $S_{x_g}(0) = 0$, and by using a wave number step size that is sufficiently large. At very high wave number, the spectral density for the ground curvature blows up. However, a tire would smooth out these high-wave number disturbances. One form of the ground height spectral density that accounts for a finite tire contact patch is

$$S_{x_g}(\kappa) = C\kappa^{-N}\left[\frac{\sin(\kappa l/2)}{\kappa l/2}\right]^2 \quad (6)$$

As can be seen from Eqs. (3) and (4), the load and damping of the aircraft depend on its ground speed V; also, the loading depends on the integral of ground speed (s, distance traveled) and the derivative of ground speed (a, aircraft horizontal acceleration). Therefore, some model for ground speed must be assumed. The ground speed model for aircraft landing used here makes several assumptions. First, the drag on the aircraft is neglected; the aircraft's velocity changes are assumed to result from braking alone. The braking force is assumed to depend only on the aircraft's lift and weight, not on local ground surface profile. Finally, the attitude of the aircraft and control surfaces is assumed to be constant during the landing roll. With these assumptions, the ground speed is given by

$$V(t) = V_0, \quad t < t_d$$

$$V(t) = \frac{V_0}{\beta_0}\frac{\{1 + \beta_0 - (1 - \beta_0)\exp[^2\mu g(t - t_d)\beta_0/V_0]\}}{\{1 + \beta_0 + (1 - \beta_0)\exp[^2\mu g(t - t_d)\beta_0/V_0]\}}$$

$$t_d < t < t_d + \frac{V_0}{2\mu g\beta_0}\log\left(\frac{1 + \beta_0}{1 - \beta_0}\right) \quad (7)$$

$$V(t) = 0, \quad \text{otherwise}$$

A different velocity model would be used for takeoff or taxi analysis.

Random Matrix Method

There are four steps in the random matrix method. First, the equations of motion for the system are cast into matrix form. Then, statistics are obtained for the matrix and load vector elements. Next, statistics of elements of the inverse matrix are found from statistics of the original matrix. Finally, the desired response statistics are computed and are used to generate further output, if needed.

The first step of the random matrix method, discretization, is fairly simple. The vibration differential equation of vibration motion for the aircraft is given by

$$m\ddot{x}(t) + c(t)\dot{x}(t) + kx(t) = f(t) \quad (8)$$

where overdots denote temporal derivatives. This equation can be discretized in the time domain through the use of central difference equations. Once this discretization is accomplished, the differential equation becomes a matrix equation

$$[A]\{\boldsymbol{x}\} = \{\boldsymbol{p}\} \quad (9)$$

where the elements of matrix $[A]$ are given by

$$\begin{aligned} A_{11} &= 2m/\Delta t^2 \\ A_{ii} &= m/\Delta t^2 + c_i/(2\Delta t), \quad i > 1 \\ A_{i(i-1)} &= k - 2m/\Delta t^2 \\ A_{i(i-2)} &= m/\Delta t^2 - c_i/(2\Delta t) \\ A_{ij} &= 0, \quad \text{otherwise} \end{aligned} \quad (10)$$

where a subscript index i is used to denote the ith discrete time. The load vector elements, incorporating both load effects and initial condition effects, are

$$\begin{aligned} p_1 &= f_0 - (k - 2m/\Delta t^2)x_0 + (2m/\Delta t - c_0)v_0 \\ p_2 &= f_1 - [m/\Delta t^2 - c_1/(2\Delta t)]x_0 \\ p_i &= f_{i-1}, \quad i > 2 \end{aligned} \quad (11)$$

where for a landing analysis, x_0 is zero and v_0 is taken to be the descent velocity V_d.

The next step in the random matrix method is to generate statistics for properties, load, and initial conditions. The complicated velocity model in Eq. (7) makes it extremely difficult to compute these statistics directly from the random variables in the problem. Therefore, Latin hypercube sampling (discussed in the following section) will be used to simulate the random variables so that the needed statistics can be computed quickly and accurately. Note that the ground field is not simulated, but the statistics of the field are used directly. The ground field has mean derivative and curvature zero, so that the expected value of the load is independent of the ground surface

$$E[f_i] = E[W]E\left[1 - \beta_0\left(\frac{V_i + V_w}{V_0 + V_w}\right)^2\right] \quad (12)$$

Similarly, cross-covariances between properties and load will not refer to the ground field, because the properties do not correlate with the ground field and because the ground slope and curvature have zero mean. However, the ground field autocovariance will be used to obtain the load's autocorrelation function:

$$\begin{aligned} E[f_if_j] = {} & E[W^2]E\left\{\left[1 - \beta_0\left(\frac{V_i + V_w}{V_0 + V_w}\right)^2\right]\right. \\ & \times \left.\left[1 - \beta_0\left(\frac{V_j + V_w}{V_0 + V_w}\right)^2\right]\right\} + E[m^2]E[R_{x_g''}(s_j - s_i)V_i^2V_j^2] \\ & + E[m^2]E[R_{x_g'}(s_j - s_i)a_ia_j] + \frac{1}{2}\rho SC_{L,\alpha}E[m]E[R_{x_g'}(s_j - s_i) \\ & \times (a_iV_j^2 + V_i^2a_j)] + \frac{1}{4}\rho^2S^2C_{L,\alpha}^2E[R_{x_g'}(s_j - s_i)V_i^2V_j^2] \end{aligned} \quad (13)$$

The expected values $E[\,]$ in Eq. (13) can be obtained with the aid of Latin hypercube sampling, as mentioned previously. Values of a, V, and s at each time step can be found for a given realization of the problem's random variables. The ground field autocovariance at certain points can be obtained by a discrete Fourier transform of the spectral density found in Eq. (6):

$$R_{x_g'}(i\Delta s) = \sum_{-N}^{N} S_{x_b}(j\Delta\kappa)j^2\Delta\kappa^3 \cos\left(\frac{2\pi ij}{2N+1}\right) \tag{14}$$

$$R_{x_g''}(i\Delta s) = \sum_{-N}^{N} S_{x_b}(j\Delta\kappa)j^4\Delta\kappa^5 \cos\left(\frac{2\pi ij}{2N+1}\right) \tag{15}$$

where

$$\Delta s = \frac{2\pi}{\Delta\kappa(2N+1)} \tag{16}$$

Often, the argument of the ground autocovariance functions will not be an integer multiple of Δs because of velocity nonstationarity. Values of the ground field autocovariance at distances other than an integer multiple of Δs should be found by interpolation, not by substituting the desired value of s into Eqs. (14) or (15). This is because higher wave numbers will dominate the curvature spectral density over a fairly large wave number range (to 10 rad/in. or higher), and so the computed curvature autocovariance will be highly oscillatory, with peaks at integer multiples of Δs. Thus, interpolation should be used for autocovariance values at other locations.

The third step in the random matrix method is the acquisition of deflection statistics from the statistics of the property matrix and load vector. To do this, the random matrix method employs the Neumann expansion. First, $[A]$ is separated into a mean and a deviatoric part

$$[A] = [\bar{A}] + [A'] \tag{17}$$

where the prime denotes the deviatoric part and the overbar denotes the mean. Then, the inverse of $[A]$ may be expressed using the Neumann expansion[17]

$$\begin{gathered}[A]^{-1} = ([I] - [P] + [P]^2 - \cdots)[\bar{A}]^{-1} \\ [P] = [\bar{A}]^{-1}[A']\end{gathered} \tag{18}$$

Note that the Neumann expansion does not require the inverse of the random deviatoric part of the matrix, though the inverse of the deterministic mean part is needed. Thus, the Neumann expansion allows computation of statistics in the inverse matrix from statistics in the original matrix.

Finally, the desired response statistics can be computed from the load element statistics and the statistics of the inverse matrix elements, using

$$\{\boldsymbol{x}\} = [A]^{-1}\{\boldsymbol{p}\} \tag{19}$$

This matrix equation has some characteristics that may speed up computation time. The deviatoric matrix is banded with a width of three elements, and the inverse mean matrix will be lower triangular. If there are n time steps, the entire random matrix technique will take on the order of n^3 multiplications, yielding a relatively fast solution. In addition, stochastic stability problems that were present in the frequency domain[14,15] are absent in the time domain formulation. Thus, the random matrix technique is ideal for obtaining statistics of the strut vibration in a light aircraft on takeoff, landing, or taxi.

Monte Carlo Techniques

There are two reasons why Monte Carlo simulation is used in this problem. First, as mentioned previously, Monte Carlo simulation of the problem's random variables is necessary to obtain the required property and load means and covariances required by the random matrix method. Also, a full Monte Carlo analysis will provide results that can be compared to those obtained by the random matrix method, so that the accuracy of each method can be examined. This Monte Carlo analysis will use a hybrid approach: the random parameters in the problem will be simulated by Latin hypercube sampling,[11,12] and the ground process will be simulated using the spectral representation method.[8,9]

Latin hypercube sampling works well at simulating a small number of correlated or uncorrelated variables with any marginal probability distribution functions, and so will be used to simulate the random parameters in this problem. The method simulates each variable with a set of ordered samples, allowing the simulated variables to match both target marginal probability density functions for each variable (from good sample generation) and covariances between variables (from good sample ordering). Note that an improved Latin hypercube formulation has recently been developed by the authors; this formulation has superior sample generation and sample ordering compared to conventional approaches,[18] and though it takes more computation time than current methods, it is far more accurate. Therefore, the new formulation will be used in this work.

Latin hypercube sampling works well for a relatively small number of random variables, but not for simulating random processes or fields. To simulate the stationary ground field, the nominal spectral representation approach will be used. The ground field height can be represented by[8,9]

$$x_g(i\Delta s) = \sum_{-N}^{N} [\sqrt{2S_{x_g}(j\Delta\kappa)\Delta\kappa}]\cos\left(\frac{2\pi ij}{2N+1} + \phi_j\right) \tag{20}$$

where Δs is defined in Eq. (16), and the ϕ_j are independent, uniformly distributed phase angles on $[0, 2\pi]$. Thus, a sample ground field can be obtained by a set of sample phase angles.

Values of ground height at distances other than integer multiples of Δs can be obtained either by interpolation or by plugging the desired distance into Eq. (20) in place of $i\Delta s$. It is not clear which method would yield more accurate load statistics. The problems with simply plugging in the distance have been mentioned in the previous section: because high wave number values of the Fourier transform of the ground curvature are dominant, the simulated ground field will be highly oscillatory between integer multiples of Δs. On the other hand, interpolating the ground field values can also lead to significant errors in ground field autocovariance. For example, when computing the variance of ground curvature at a point halfway between integer multiples of Δs, the computed variance from interpolation would be

$$R_{\text{est}}(0) = \tfrac{1}{2}R_{x_g''}(0) + \tfrac{1}{2}R_{x_g''}(\Delta s) \tag{21}$$

when the actual variance is, of course, $R_{x_g''}(0)$. The computed variance will be too small, because $R_{x_g''}(0)$ is larger than $R_{x_g''}(\Delta s)$. Because it is not clear which approach gives better results, both interpolation and plugging in will be used in the hybrid Monte Carlo simulation results for this problem.

When using the hybrid Monte Carlo technique, Latin hypercube is used to simulate random parameters in the problem, while the random phase angles for the ground process are obtained by a random number generator. Note that multiple ground fields are needed for each Latin hypercube sample set to ensure accurate results.

Details of Computation

Random vibration response will be estimated for two light amphibious aircraft: 1) the Teal Amphibian and 2) the Lake Renegade. The two aircraft differ primarily in size: the Teal seats two, whereas the Lake seats four. Data for the Teal Amphibian were found in an aircraft design book,[19] whereas data for the Lake Renegade were obtained from Lake Aircraft's World Wide Web site. Deterministic parameters for each aircraft are specified in Table 1, whereas the random parameters used for the computation, which are independent, are specified in Table 2; the descent velocity distribution was obtained from an aircraft design manual.[20] Three ground surfaces are used in this work; all are assumed to have spectral densities of the form found in Eq. (6), with N equal to 2.1, and with various values of C, which are found in Table 3.[16]

The response of both aircraft landing on the three surface types will be evaluated. For some cases, the random parameters in the problem are as shown Table 2; for others, these random parameters are taken to be their mean values. In all cases, the time step is 0.01 s, and there are 300 time steps considered, so the first 3 s of the landing are analyzed; the time step allows roughly 20 time steps per natural period of either aircraft. Also, the wave number step size is 0.0000785 rad/in. (which is sufficiently large to avoid problems at the first wave number step), and 500 wave number steps are used, yielding a maximum wave number of 0.03925 rad/in. This maximum wave number seems rather small, but it is adequate to bracket the natural frequency of either aircraft at typical landing speeds, and it is large enough to avoid low-wave number problems for the ground spectral density. For each case, 100 hypercube samples were used, and 10 ground fields were used for each hypercube sample in the hybrid Monte Carlo simulation. The random matrix runs took roughly twice as long as the Monte Carlo runs, mainly because of computation of input statistics.

Table 1 Deterministic parameters used in this work[a]

Symbol	Teal value	Lake value
$C_{L\alpha}$	4.864	5.034
E, lb/in.^{2}	3.00E+7	3.00E+7
g, in./s^2	386.4	386.4
h, in.	0.69	1.1875
I, in.^{4}	0.082	0.66
l, in.	3.0	3.0
L, in.	22.5	40
S, ft^2	157	164
γ	0.08	0.08
θ, deg	58	58
ρlb s^2/in.^{4}	1.13E−7	1.13E−7

[a]Values of these parameters are shown for the Teal Amphibian and the Lake Renegade.

Table 2 Random variables used in this work[a]

Symbol	Type	Mean	Standard development
$t_{,d}$ s	Rayleigh	0.5	0.261
V_O, in./s	Normal	881/1057	70.4
V_d, in./s	Weibull	14.46	9.178
V_w in./s	Normal	175	70.4
W, lb	Normal	1975/2600	90/200
β_O	Normal	0.75	0.05
μ	Normal	0.7	0.04

[a]Numbers with slashes are values for the Teal Amphibian/Lake Renegade aircraft.

Table 3 Surface spectral density coefficients for this work

Surface type	C, in.$^{0.9}$ rad$^{1.1}$
Smooth highway	8.48E−5
Gravel roadway	7.77E−4
Rough runway	1.63E−3

Results

To compare and evaluate the proposed methods, a Teal aircraft with random properties landing on a rough runway was examined by the random matrix method and by the Monte Carlo hybrid method with plugging-in and interpolation for the ground field. The mean value of response (as a function of time) for each case is found in Fig. 2, whereas the response variances are plotted in Fig. 3. Note that there are large discrepancies between the curves for the Monte Carlo methods (and it is not clear which Monte Carlo method should be considered correct), and that the random matrix variance is nearly halfway between the Monte Carlo variances. Clearly, Monte Carlo analysis is unsuitable for this problem, primarily because interpolation of ground statistics (used by the random matrix method) is superior to interpolation of ground sample functions, which are used by Monte Carlo methods. Thus, the random matrix method can be considered more accurate than the Monte Carlo hybrid methods for this problem and can be used to investigate the behavior of system response and landing gear fatigue.

The effects of surface type, aircraft, and property randomness can now be examined. First, the mean response for each aircraft with random or deterministic properties is plotted in Fig. 4; the mean response is independent of terrain type. The Teal has a higher mean deflection than the Lake aircraft, and the results for deterministic properties vary somewhat from those for random properties. The effects of surface type on response variance can be seen by analyzing a Teal aircraft with

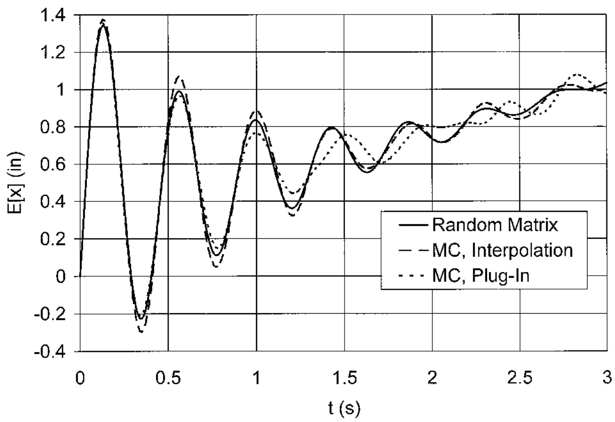

Fig. 2 Mean response obtained by various techniques for a Teal aircraft with random properties landing on a rough runway.

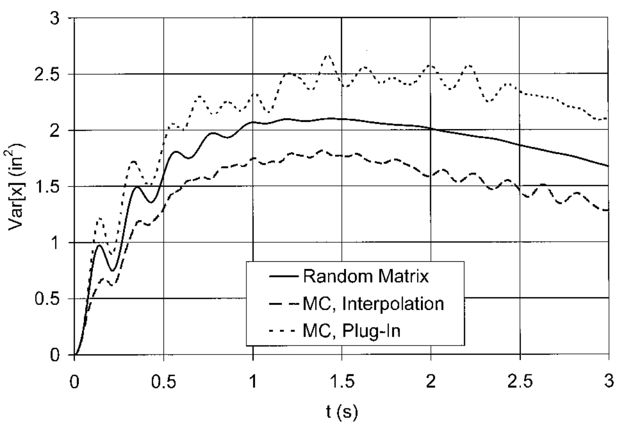

Fig. 3 Variance of response obtained by various techniques for a Teal aircraft with random properties landing on a rough runway.

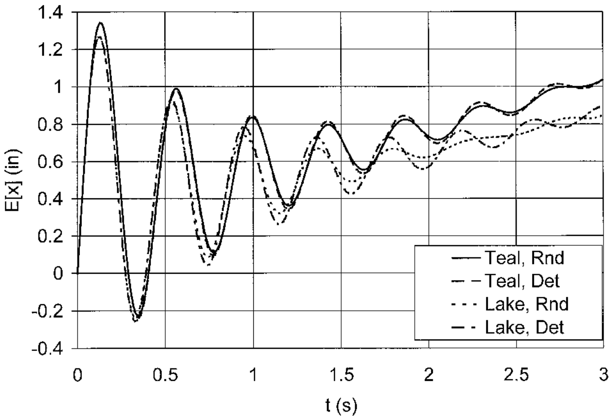

Fig. 4 Mean response for each aircraft with random or deterministic properties.

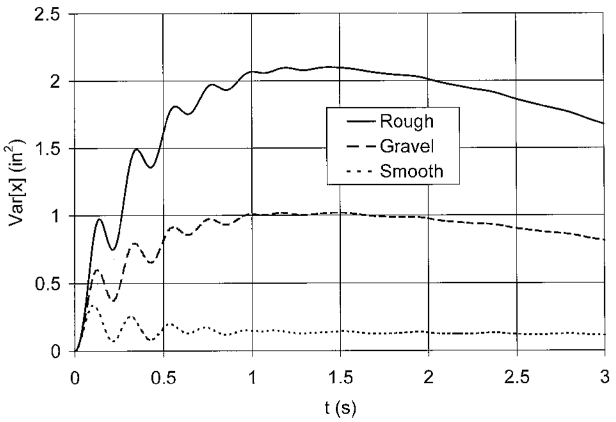

Fig. 5 Effects of terrain on response variance of a Teal aircraft with random properties.

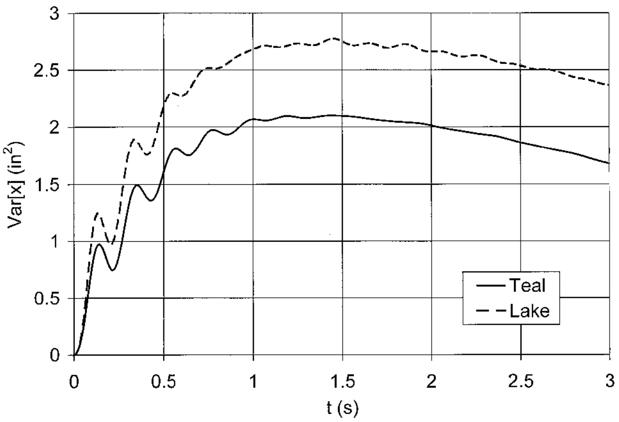

Fig. 6 Response variance for each aircraft with random properties landing on a rough runway.

random properties landing on all three terrain types; the response variance for each terrain type can be found in Fig. 5. Of course, rougher surfaces give higher response variances, as would be expected. For the rough road, the variance is often larger than the mean response squared, and so it is likely that the aircraft would leave the ground when landing on this surface. However, the response variance for all landing surfaces is sufficiently large that response randomness cannot be neglected in this problem.

To see the difference between the responses of the different aircraft, each aircraft with random properties is analyzed on a rough road; the corresponding response variances are shown in Fig. 6. Note that the response variance for the Lake aircraft

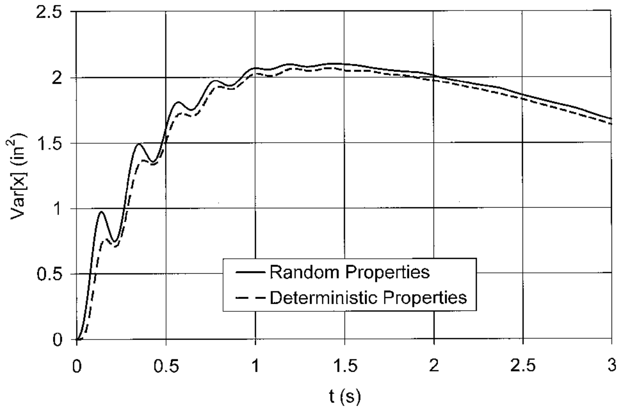

Fig. 7 Property randomness effects on response variance of a Teal aircraft landing on a rough runway.

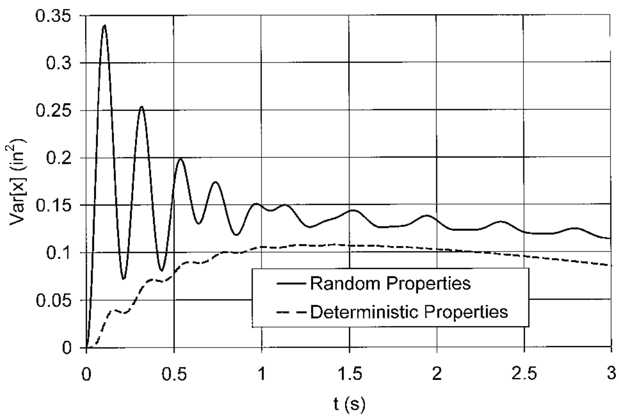

Fig. 8 Property randomness effects on response variance of a Teal aircraft landing on a smooth runway.

is higher than that for the Teal. Finally, the effects of property randomness on variance for a rough landing surface are found in Fig. 7; these effects for a smooth surface are shown in Fig. 8. It is clear from Figs. 7–8 that for rough landing surfaces the surface randomness is dominant in the response, whereas for smooth surfaces, the property randomness is more important. Thus, the importance of property randomness is dependent upon landing surface, and for rough surfaces, the property randomness may be neglected, allowing a simpler analysis.

Conclusions

Two approaches for investigating nonstationary random parametric vibration were presented: 1) a time-domain random matrix method, which used Latin hypercube sampling to obtain difficult input statistics; and 2) a Monte Carlo hybrid approach, which combined Latin hypercube sampling and spectral representation. Because the spectral representation method admitted two approaches to finding values of ground field slope and curvature at values other than at integer multiples of the space step size, interpolation, and plugging-in, two Monte Carlo hybrid methods could be examined. These approaches were used to examine random response in light aircraft landing gear during a landing. A vibration model of the aircraft was presented with a simple nonstationary velocity model, yielding a nonstationary random parametric vibration problem.

The random matrix method and the hybrid Monte Carlo methods were used to examine two different light amphibious aircraft with random or deterministic properties landing on three terrain types. The hybrid Monte Carlo response results varied widely from each other, and so Monte Carlo analysis was considered unsuitable for this problem; the random matrix method was considered accurate. Response randomness could not be neglected in this problem because the response variances were significant compared to the square of the mean response values. The response results depended heavily upon the aircraft and the ground surface; property randomness did not seem to have much effect in this problem for rough runways (and can be safely neglected, yielding a simpler analysis), but it did have a more noticeable effect for smoother landing surfaces.

The time-domain random matrix method is a fast, reliable, flexible, and accurate computational technique for nonstationary random parametric vibration analysis. Because the random matrix method requires property and load statistics as input, in some cases (such as the problem studied here) it is difficult to compute these statistics; in this case, Latin hypercube sampling should be employed to obtain these statistics. On the other hand, where possible, statistics should be used directly, as was done for the ground surface in this work. Finally, note that the random matrix technique can be extended to multiple-degree-of-freedom systems in a straightforward manner. Thus, the time-domain random matrix method is recommended for nonstationary random parametric vibration analysis of linear systems.

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